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Late Pleistocene geomagnetic excursion in Icelandic lavas: confirmation of the Laschamp excursion

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In 1980 Kristjansson and Gudmundsson [1] reported a late glacial geomagnetic excursion in three hills in the Reykjanes peninsula, Iceland, with shallow negative inclinations and westerly declinations. They named it the Skalamaelifell excursion. More extensive field work has identified the same excursional paleomagnetic direction (declination = 258° , inclination = -15°) at four additional outcrops in a 10×10 km area in the Reykjanes peninsula. The excursion lavas are olivine tholeiites with similar petrography and chemical compositions. Paleointensity determinations by the Thellier method average $4.2 \pm 0.2 \mu$ T for 8 samples, more than an order of magnitude weaker than the present geomagnetic field in Iceland. Together, these results suggest extrusion of the excursion lavas in a very brief span of time, probably less than a few hundred years.

K-Ar dating of the excursion lavas gives a mean age for 19 determinations of 42.9 ± 7.8 ka (2σ) . Compilation of thirty K-Ar ages of the Laschamp and Olby flows by three laboratories yield a new age for the Laschamp excursion in France of 46.6 ± 2.4 ka (2σ) . The age of the excursion in southwestern Iceland is statistically indistinguishable from the Laschamp excursion at the 95% confidence level, and both have very low paleointensities. Therefore, we suggest that the Laschamp and Olby flows in France and the Skalamaelifell units of Iceland recorded essentially the same geomagnetic excursion. Differences in the virtual paleomagnetic poles (VGPs) of these excursions may be due to (1) the probable non-dipole character of the geomagnetic field during the excursion, (2) rapid geomagnetic secular variation and possible small age differences of the extrusive rocks in France and Iceland, and/or (3) crustal magnetic anomalies which might dominate the local geomagnetic field during the excursion at either or both locations.

1. Introduction

Geomagnetic polarity time scales for the last few million years [2,3] show the Brunhes normal polarity chron of the past 0.72 million years (m.y.) to be 2–3 times longer than the average length of polarity intervals for the last 40 m.y. [4]. However, evidence has been accumulating for the existence in the Brunhes of several short-lived polarity reversals or excursions, including Big Lost at about 0.6 Ma [5], the Emperor at about 0.5 Ma [5–7], and the Blake at about 0.11 Ma [5,8–11].

In the most recent 0.1 m.y. several possible paleomagnetic excursions have been detected, including the Mono Lake excursion from 26,000 to 29,000 years before present (ka) [12–14], the Lake Mungo excursion at 30-32 ka [15,16], an excur-

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sion at circa 17 ka [17-19], and the Laschamp excursion between 40 and 50 ka [20-23]. Of these excursions only the Laschamp has been identified in igneous rocks.

Geomagnetic reversals and excursions preserved in sedimentary and igneous sequences can be valuable for stratigraphic correlations, especially for the Neogene. In addition, detailed studies of the geomagnetic field for the Brunhes chron, for which high-resolution records are more readily available, are important for understanding the full spectrum of geomagnetic behavior.

The paleomagnetic direction of the Laschamp and Olby units in the Massif Central in France can be considered to have reversed polarity. However, recent measurements [24] have shown that the flows recorded a very low paleointensity (less than 15% of the present field), indicating that the Laschamp should be more properly classified as a geomagnetic excursion. An independent confirmation of the Laschamp excursion has been wanting, but, recently, a geomagnetic excursion corresponding in age to the Laschamp has been observed in marine sediments from the Gulf of California [25].

Kristjansson and Gudmundsson [1] reported paleomagnetic evidence for a "geomagnetic excursion in late-glacial basalt outcrops in southwestern Iceland". In this paper we present paleomagnetic results with the same excursion direction from additional outcrops in this area of the Reykjanes peninsula. Furthermore, geomagnetic paleointensity results and K-Ar age determinations of the excursion lavas show that they are contemporaneous with the Laschamp excursion.

2. Regional setting

The Reykjanes peninsula of southwestern Iceland (Fig. 1) is about 60 km long and 20 km wide. It is mostly an arid plain, rising from a mean altitude of 50 m in the west to 200 m east of the Blafjoll ridge. The plain is covered by recent lava flows and volcanic detritus.

Numerous volcanic hills and ridges rise above this plain, especially in the southern part of the



Fig. 1. Map of sampling area in the Reykjanes peninsula, southwestern Iceland. Shaded zones indicate location of excursion sites; filled circles represent units with normal directions.

peninsula. Geological and geophysical evidence suggest that this is the site of the active axial volcanic zone of the Mid-Atlantic Ridge. The relief of most of these hills is of the order of 100 m. Some hills, lava shields and minor craters have been volcanically active in postglacial time, and these have produced most of the lavas covering the peninsula, while other lavas have originated in fissure eruptions. There is some evidence of episodicity in the eruptive activity, with an approximate time interval between episodes of about 1000 years. The last of these occurred in 900–1400 A.D., involving the entire length of the Reykjanes peninsula [26, table 1].

The postglacial production of the two westernmost fissure swarms in the volcanic zone of the peninsula has been estimated at 13–14 km³ of lava [27]. This activity corresponds to a few meters of additional lava cover per thousand years, and is apparently accompanied by graben subsidence of similar magnitude.

Many of the hills and ridges on the Reykjanes peninsula, protruding through the postglacial cover, belong to older volcanic cycles. It is considered that most of these hills came into being during the last glacial period, either as submarine or subglacial eruptions [28]. They are composed of hyaloclastites, breccia, and pillows, with sporadic lava flows occurring mostly at the higher elevations in each hill. At the top of some of the hills, remains of craters can be seen. The products of these extinct volcanic edifices are heterogeneous and seldom overlap each other in space, hence it has been difficult to determine their stratigraphy and relative ages.

3. Petrology of excursion basalts

Lava samples for petrological examination were obtained from the five hills where the excursion was identified: Siglubergshals, Skalamaelifell, Höfdi and the two Hraunssels-Vatnsfell (Fig. 1). These hills are made up of hyaloclastites capped by subaerial lava flows or their remnants. The hills are therefore small tuyas (Icelandic; stapi). The ice cover was thin at the time of the eruptions, of the order of 80-150 m above the present level of the lava plains of the peninsula. The volumes of these eruptions were small, and the total volume of the exposed excursion extrusives is estimated at 0.2-0.3 km³. Accounting for material removed by glacial action, however, the original volume may have been considerably larger.

TABLE 1

Locality:	Sigluberg	gshals		Skalamae	elifell	Höfdi	Vestara	Austara		
Locality: Analysis No.: SiO ₂ TiO ₂ Al ₂ O ₃ Fe ₂ O ₃ FeO MnO MgO CaO Na ₂ O K ₂ O P ₂ O ₅ Cr ₂ O ₃ LOI Total	lava	lava	lava	lava	dyke	lava	Hraunssels-	Hraunssels-Vatnsfell		
	11700	11703	11704	11705	11706	11707	Vatnsfell lava 11708	lava 11710	lava 8617	
SiO ₂	47.32	47.11	46.70	47.10	47.07	47.21	47.13	46.11	46.80	
TiO ₂	1.81	1.81	1.72	1.89	1.82	1.94	1.87	1.86	1.89	
Al_2O_3	15.46	15.41	14.43	14.50	14.74	14.79	15.18	14.22	13.93	
Fe ₂ O ₃	1.63	1.50	2.10	1.83	1.78	1.78	1.73	1.76	2.18	
FeO	9.92	10.20	9.86	9.95	9.81	10.04	10.13	10.19	10.05	
MnO	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.20	
MgO	9.14	9.39	11.26	10.30	10.35	9.74	9.24	10.74	11.32	
CaO	11.92	11.86	11.29	11.73	11.80	11.93	11.98	11.45	11.43	
Na ₂ O	1.82	1.81	1.73	1.73	1.69	1.77	1.83	1.69	1.71	
K ₂ O	0.17	0.17	0.16	0.24	0.19	0.22	0.17	0.19	0.18	
P_2O_5	0.20	0.20	0.20	0.21	0.19	0.20	0.20	0.20	0.20	
Cr ₂ O ₃	0.06	0.07	0.11	0.09	0.10	0.08	0.06	0.10	0.11	
LOI	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	
Total	99.65	99.73	99.76	99.77	99.74	99.90	99.72	99.71	100.01	

Rock compositions are olivine tholeiites. Analyses were performed by S.T. Ahmedali, McGill University, Montreal, by X-ray fluorescence on fused beads prepared from ignited samples; FeO was determined by titration.

The lava flows are compact, gray basalts, and most are between 2 and 4 m thick. Samples were collected at the paleomagnetic sites; additional samples were also obtained to gain broader coverage of the extrusives in each hill. Scoriaceous or visibly oxidized zones were avoided. The lavas are generally very fresh as indicated by the average Fe_2O_3/FeO (weight percent) ratio of 0.18 and low content of volatiles in the samples that were chemically analyzed.

Petrographically the extrusive rocks from the five hills are all very similar. They contain up to 15 vol.% of olivine phenocrysts, which commonly carry inclusions of chromian spinel. Plagioclase phenocrysts are ubiquitous and may form up to 12 vol.% of the rocks. The groundmass is composed of plagioclase, clinopyroxene, titanomagnetite and ilmenite. No hydrothermal alteration and only minor deuteric oxidation could be detected in the microscope examination.

Major element analyses of samples from eight lavas and one dike are presented in Table 1. Rock compositions are all olivine tholeiites with a quite high content of MgO. It is possible that the samples with the most MgO (11.26 and 11.32 wt.% MgO) are slightly olivine cumulative. As regards major element composition, the excursion rocks are very similar to the olivine tholeiite lava shields of the western Reykjanes peninsula, which probably erupted about 9–11 ka [27,29].

The petrographical and chemical similarity of the described rocks, as well as the proximity of the outcrops, might suggest that the five hills were produced during a single volcanic eruption event. However, it is not possible to prove this by petrochemical data, because it is also possible that different eruptive episodes in the same restricted volcanic area may be of nearly identical chemical composition.

4. Related paleomagnetic studies in Iceland

Peirce and Clark [30] described paleomagnetic results from a site of reversely magnetized pillows in the central volcanic zone at Thingvallavatn Lake, some 60 km ENE of our area. This site is rather poorly exposed at the top of a small hill. The various pillows yielded reasonably stable but somewhat scattered remanence directions. A few tens of meters lower on this hill, there are normal polarity basalt outcrops. We sampled several exposures in nearby hills but failed to find other reversed units. The pillows of this "Maelifell event" are very porous olivine-rich basalts, and they are not well suited for K-Ar dating (Y. Cornette, personal communication, 1982).

Kristjansson and Gudmundsson [1] described laboratory magnetic measurements on oriented rock cores, mostly from lava flows, which were collected from hills on the Reykjanes peninsula. Several of these lavas had previously yielded apparent reverse magnetic polarity upon testing of hand samples by a portable fluxgate magnetometer. Alternating field (AF) demagnetization of specimens showed that in some cases they had been remagnetized by lightning. In three separate hills, however, a primary remanence having similar shallow negative inclination and westerly declination was isolated. Such directions are generally very rare in Icelandic rocks [31]. It was therefore logical to conclude that the three hills (Siglubergshals, Eastern Hraunssels-Vatnsfell, and Skalamaelifell) belong to a common, now extinct, fissure swarm, which produced at least these three hills within a short time span (compared to historical geomagnetic secular variation time scales). Kristjansson and Gudmundsson [1] named this geomagnetic feature the "Skalamaelifell excursion".

5. Paleomagnetic sampling and stratigraphy

Further sampling for paleomagnetic measurements was undertaken from 1982 to 1987 in the three hills studied by Kristjansson and Gudmundsson [1] and their vicinity. This has led to the discovery of additional outcrops belonging to the Skalamaelifell excursion (Fig. 1), and some stratigraphic relationships have emerged from these studies.

(1) Excursion directions were found in lavas capping Bratthalskrokur and Einihlidar, low ridges lying NNE of Skalamaelifell. They also occur in isolated lava outcrops on the slopes of the Langihryggur ridge NNW of Skalamaelifell. Some of the hyaloclastites at these locations also seem to carry a primary remanence having the excursion direction (L. Kristjansson, unpublished data; B.B. Ellwood, personal communication, 1979), but they are poorly consolidated and have very weak remanence intensity, so that sampling and measurements of these rocks are only in preliminary stages.

(2) The excursion also occurs in lavas just north of the top of the ridge Höfdi, east of Skalamaelifell. In the southern (and clearly older) part of Höfdi there are normal polarity feldsparphyric lavas, and a dike which is also of normal polarity.

(3) The hill Eastern Hraunssels-Vatnsfell has now been sampled more extensively than before. It is confirmed that all lavas in this hill belong to the excursion, but it is cut by a normal polarity dike. In the nearby Western Hraunssels-Vatnsfell, we discovered lava flows and a sill-like intrusion into hyaloclastite, which record the Skalamaelifell excursion. (The lava exposures in the northern part of this hill have apparently been subjected to post emplacement movements, but they most likely belong to the same episode.)

(4) Excursion lavas from the Siglubergshals ridge have filled up a channel between the Fiskidalsfjall and Festarfjall hills, both of which are composed mostly of hyaloclastites. There are excellent exposures of at least eight successive excursion lava units in a gully south of the road, some showing pillow structures.

(5) In several hills and minor outcrops of basalts, protruding through the postglacial lava cover in the area of Fig. 1, only normal polarity lavas have been found so far. These hills include Driffell, Keilir, Fiskidalsfjall, Festarfjall/Lyngfell, Borgarfjall, Kistufell, Sandfell, Storihrutur and Slaga. Sampling in the area is, however, still very fragmentary.

6. Paleomagnetic directions

All specimens were AF demagnetized at 10 and 20 mT peak fields. Specimens whose remanence direction changed by more than 5° during the latter treatment were also demagnetized at 25 mT peak field, but those clearly affected by lightning were rejected.

Fig. 2 shows the stable directions of the 97 excursion samples, all with a stable remanence and an intensity exceeding 0.06 A/m after the 10 mT AF treatment. Following Kristjansson and Gudmundsson [1], the data were grouped in the three geographical areas, and they include the 55 samples of the former study. About 50 different volcanic units may be represented, but it is often



Fig. 2. Southwestern segment of stereogram showing stable directions of the excursion samples. Geographic groups A, B, C are identified in Table 2. Circles, squares, triangles correspond to groups A, B, C, respectively; larger symbols are the mean directions of respective groups. Open (closed) symbols refer to upper (lower) hemisphere directions. Diamond is the mean direction of the normal samples.

difficult to trace individual units in the discontinuous outcrops typical in this region.

Table 2 shows the mean paleomagnetic directions of the three sample groupings, also shown in Fig. 2. There is no significant change of the mean

TABLE 2

Average paleomagnetic directions for the Skalamaelifell excursion lavas

Sites ^a	Ν	D	Ι	k	θ_{63}	α ₉₅	
Group A	44	261.2	- 20.7	100.4	8.1	2.2	
Group B	26	254.7	-12.4	39.2	12.9	4.6	
Group C	27	258.3	-10.0	28.9	15.1	5.3	
Groups $A + B + C$	97	258.6	-15.5	40.0	12.8	2.3	
Groups $A + B + C$	3	258.0	-14.4	158.9	6.4	9.8	
Normal samples	100	36.3	75.5	28.3	15.2	2.7	

^a Group A: Siglubergshals; group B: Austara and Vestara Hraunssels-Vatnsfell and Höfdi; group C: Skalamaelifell, Langihryggur, Bratthalskrokur and Einihlidar; normal: pre-Holocene mostly.

N = number of cores, usually 1 to 2 per outcrop; D = declination, I = inclination, of the stable remanence, in degrees; k = precision parameter, and α_{95} , the angular radius of the 95% cone of confidence, of the associated Fisher distribution [41]. θ_{63} , the angular standard deviation.

paleomagnetic directions from the results of Kristjansson and Gudmundsson [1]. The excursion directions cluster around a westerly declination with a shallow negative inclination. No discernible path of the excursion directions can be traced for the units samples so far, and it is likely that they were all emplaced during a single spate of eruptions lasting on the order of 100 years, consistent with their occurrence at similar elevations.

In addition to geomagnetic secular variation some of the scatter of the directions around this mean is undoubtedly due to slight block movement, frequently seen at the edges of the lava outcrops (caused by frost and other mechanical weathering agents). Another significant part of the scatter might be the result of local magnetic anomalies, originating in underlying strongly magnetized basalts, which contributed to the geomagnetic field at the time of emplacement. A third factor that might cause directional scatter is the presence of secondary magnetization components, whose influence on the directional measurements might be relatively greater here, due to the low intensity of the primary remanence.

The mean paleomagnetic direction of the normal polarity flows from this area (Fig. 1; Table 2) is somewhat right-handed. However, these normal lavas probably do not represent sufficient time to express the long-term average behavior of the geomagnetic field in the Brunhes chron, and they recorded no major excursions of the field. The reader is referred to Kristjansson and McDougall [31], and Kristjansson [32] for comprehensive statistical analysis of the virtual geomagnetic pole dispersion and apparent field strength in Iceland in the Neogene.

7. Magnetic properties

The low field susceptibility of the excursion flows and those with normal directions in this area are shown as histograms in Fig. 3. The geometric means of the susceptibility of these assemblages are very similar at 7.3 and 8.9×10^{-3} SI volume units, respectively. Histograms of the remanence intensity after AF demagnetization to 10 mT are also shown in Fig. 3. The geometric mean remanence intensity of 112 excursion specimens is 0.24 A/m, while for the 100 specimens from normal polarity late glacial lavas in the area of Fig. 1 the



Fig. 3. Histograms of low field susceptibility (χ_0) and magnetization intensity (M_{10}) of excursion (white) and normal polarity samples (shaded from late glacial units in the same region of the Reykjanes peninsula. χ_0 is in SI volume units, and M_{10} (A/m) are NRM intensities AF demagnetized to 10 mT to remove secondary remanences. The table shows the arithmetic and geometric means of χ_0 and M_{10} .

Polarity	N	χ_0	M_{10}	Mean
Skalamaelifell excursion	112	0.0073 0.0097	0.239 0.384	geometric arithmetic
Normal	100	0.0089 0.0111	3.11 4.15	geometric arithmetic

N = number of specimens measured.

geometric mean is 3.1 A/m. These data provide a preliminary indication that the geomagnetic field during the excursion episode was considerably weaker than normal polarity fields around this time.

Saturation magnetization versus temperature (J_s-T) experiments were conducted for five excursion specimens. Heatings to 600 °C were done in air at standard pressure. Three of the specimens exhibited a single Curie point (T_c) between 520 and 580 °C. The other two specimens showed T_c values from 200 to 300°C. One of the latter specimens had a nearly reversible $J_s - T$ curve, while the magnetic minerals of the other transformed on heating to a more magnetic phase with $T_c >$ 500 °C. Marshall et al. [33] reported $J_s - T$ results for six very stable excursion specimens, which had been selected for paleointensity studies. Their results show single T_c values between 555 and 570 °C. The J_s-T curves are similar for both heating and cooling, but there are increases in J_s of up to 40% after cooling. These results suggest that the predominant magnetic minerals are titanomagnetites, and that the more stable specimens, suitable for paleointensity studies, have low titanium contents, suggestive of high-temperature deuteric oxidation.

8. Paleointensity studies

For paleointensity studies we resampled outcrops at Siglubergshals, where previous measurements had shown the remanence to be stable with only minor overprinting, and from each of 20 cores one specimen was progressively AF demagnetized to 100 mT to assess the remanence stability and to determine the stable direction. Specimens for paleointensity experiments were selected from those with relatively higher AF stabilities and with minimum secondary components of magnetization. Finally, the primary criterion for choosing specimens was that the direction of their natural remanent magnetization (NRM) was closest to the stable excursion direction (see Fig. 4).

the paleointensity experiments were carried out by the Thellier double heating method [34], modified by Coe [35], where the specimens are heated to progressively higher temperatures, incrementally demagnetizing the NRM. At each temperature step the samples were heated twice for 90 minutes in a quartz tube evacuated to pressures



Fig. 4. Stereogram showing NRM directions of samples from the Siglubergshals locality, sampled for paleointensity studies. Open (closed) symbols indicate upper (lower) hemisphere directions. Star is the stable site mean direction. Diamonds represent directions of specimens chosen for paleointensity studies.

less than 10^{-5} Torr, and then cooled to room temperature. The first heating was done in null magnetic field to determine the residual NRM. The second heating was done in a known applied magnetic field to produce a laboratory partial thermal remanent magnetization (PTRM). The remanence was always measured at room temperature. The temperature reproducibility at each step was about 2°C. The results of Thellier paleointensity experiments can be conveniently displayed on a partial NRM versus partial TRM (PNRM-PTRM) diagram, where the (PNRM, PTRM) pairs are shown for the different temperature steps. Ideal behavior in the Thellier sense is achieved by linear data over the entire range of blocking temperatures, and the negative slope is proportional to the paleointensity.

Despite careful sample selection, many rocks are not suitable for reliable paleointensity determinations, because of (1) chemical alterations of the magnetic minerals during the laboratory heatings, which often irreversibly modify the capacity of PTRM acquisition, and (2) the persistence of subtle secondary magnetization components. One of the main advantages of the Thellier procedure is that unacceptable behavior can often be detected during the experiments as deviations from linearity of the data and by PTRM checks, which consist of attempts to reproduce PTRM at a lower temperature after the sample was heated to a higher temperature.

Eight specimens were selected for paleointensity studies, and the results are summarized in Table 3; PNRM-PTRM diagrams are shown in Fig. 5 together with projections of the remanence during thermal demagnetization onto horizontal and vertical planes. The applied laboratory field was 30 μ T for all the paleointensity experiments.

Table 3 lists the NRM and stable directions of the eight chosen specimens, and it is seen by comparison with group A in Table 2 that some of the specimens carry a significant secondary viscous component, probably produced in the much stronger present field. This interpretation is supported by the thermal demagnetization diagrams of Fig. 5, where it is evident that the secondary remanence is removed by 150 °C. Flattening of the PNRM–PTRM data was observed for several samples at different temperatures above about 350 °C. The increased PTRM capacity at the



Fig. 5. NRM-TRM diagrams of the Thellier paleointensity experiments together with projections of the remanence onto horizontal and vertical planes during thermal demagnetizations. There is a one to one correspondence in the data for each specimen between the NRM-TRM diagram and thermal demagnetization plot. The remanence values of the NRM-TRM diagram are normalized to the initial NRM value. Hence, the first point of the NRM-TRM diagram is defined at (NRM \equiv 1, TRM \equiv 0). In the NRM-TRM diagrams the diagonal numbers adjacent to the data points indicate the temperature in °C; the closed circles are the data used to calculate the paleointensity, and the triangles indicate PTRM checks.

LATE PLEISTOCENE GEOMAGNETIC EXCURSION IN ICELANDIC LAVAS

Skalamaelifell excursion: paleointensity determinations											
Specimen	M_0	I_0, I_s	D_0, D_s	N	T_{\min}	$T_{\rm max}$	f	g	q	$F_{\rm e} \pm \sigma$	
SK-1	0.45	11, -18	274, 275	5	150	350	0.48	0.75	4.5	3.2 ± 0.3	
SK-2	0.41	5, -18	271, 269	4	200	350	0.48	0.66	2.6	3.6 ± 0.4	
SK-5	0.54	-6, -18	264, 271	5	150	350	0.51	0.74	4.9	4.4 ± 0.3	
SK-6	0.18	-6, -2	274, 271	7	150	450	0.41	0.50	6.0	4.2 ± 0.1	
SK-7	0.14	-13, -21	279, 277	6	200	450	0.49	0.51	5.9	4.5 ± 0.2	
SK-10	0.23	25, -1	269, 269	6	200	450	0.45	0.77	1.8	4.6 ± 0.9	
SK-19	0.08	-7, -20	264, 262	6	220	450	0.42	0.74	6.7	4.8 ± 0.2	
SK-20	0.38	-13, -27	265, 259	4	350	500	0.23	0.64	2.2	5.1 ± 0.3	
					Weight	ed mean:	N = 8,			$\overline{F}_{e} = 4.2 \pm 0.2$	

TABLE 3

 $M_0 =$ NRM intensity in A/m; I_0 , $I_s =$ the NRM, stable inclinations in degrees, respectively; D_0 , $D_s =$ the NRM, stable declinations in degrees, respectively; N = number of consecutive points (temperatures) used for determining the paleointensity; T_{min} , $T_{max} =$ minimum, maximum temperatures in degrees Celsius, respectively used to determine the paleointensity; q = quality factor of paleointensity determinations for individual specimens, and f and g are ancillary parameters from [36]; $F_e \pm \sigma$ = paleointensity of excursion specimens and associated standard deviation (in μ T).

higher temperatures is probably caused by chemical and mineralogical evolution of the magnetic minerals during the laboratory heating.

The value of the quality factor (q) in Table 3 depends on several factors including the fraction (f) of the NRM and on the number (N) of PNRM-PTRM points used in the analysis [36]. For several specimens in Table 3 a considerably higher value of q could be obtained by using additional points covering a greater fraction of the NRM. For example, sample SK-1 (Fig. 5) would have q = 8.9, for the 8 data between 150 and 500 °C. Moreover, for SK-1 and for most of the samples of Table 3, the analysis with more higher temperature points would also lead to lower paleointensities along with the higher q values. However, despite higher q values, PTRM checks and curvature of the PNRM-PTRM points suggest that several of the specimens were affected by an increase in TRM capacity at the higher temperatures. Hence maximizing q might not always lead to the most correct paleointensity determinations.

The weighted mean paleointensity of the 8 specimens from the Skalamaelifell excursion is: $\overline{F}_{e} = 4.2 \pm 0.2 \ \mu T$ (the uncertainty represents 1 standard deviation of the mean). This value is indistinguishable from the value obtained by Marshall et al. [33]. A considerably lower paleointensity would have been obtained, had it been determined by maximizing q. The low paleointensity determined for the excursion units is more than an order of magnitude below the

current 52 µT geomagnetic intensity in the Reykjanes peninsula.

9. Geochronology

Precise dating of these very young volcanic rocks by the K-Ar method is limited by two factors: (1) the long half-life (small decay constant) of 40 K, and (2) the low abundance of K in the basalts. This means that only small amounts of radiogenic ⁴⁰Ar accumulate in the rock over periods of 10^4 – 10^5 years. Argon is also a major component of the atmosphere which equilibrates with the lavas as they cooled. Hence special attention must be paid to the principal correction in these age determinations, that for atmospheric argon contamination.

Cassignol, Gillot and Cornette [37,38] have discussed the factors which may limit the precise measurement of small amounts of radiogenic argon within a total ⁴⁰Ar signal composed principally of atmospheric argon, contributed both from the sample and from the extraction system. Central to their method is regular and repeated argon isotopic measurements of pipetted aliquots from a reservoir of atmosphere, to determine precisely the atmospheric ratio ${}^{40}\text{Ar}/{}^{36}\text{Ar}$ recorded by the mass spectrometer. Departures from the accepted value of 295.5 vary with time for a given instrument, but we can define the proportion of radiogenic argon $(*^{40}Ar)$ in a rock sample as:

 $*^{40}$ Ar = 40 Ar ($R_s - R_a$)/ R_s



Fig. 6. The principal correction of total ⁴⁰Ar concentration measured from fused whole rock samples to determine radiogenic ⁴⁰Ar is that for atmospheric ⁴⁰Ar from the sample and extraction system. Measurements of the atmospheric composition of argon (${}^{40}\text{Ar}/{}^{36}\text{Ar} = R_a$) are made routinely and vary smoothly over the lifetime of the source filament shown. At the time of measurement of sample argon composition (${}^{40}\text{Ar}/{}^{36}\text{Ar} = R_s$) the best-fit R_a is known to $\pm 0.1\%$. The periods of measurement of the Iceland and Laschamp samples are indicated by horizontal bars.

where R_s and R_a are the uncorrected measured ratios ${}^{40}\text{Ar}/{}^{36}\text{Ar}$ for the sample and for atmospheric argon, and ${}^{40}\text{Ar}_t$ is the total measured ${}^{40}\text{Ar}$ concentration. Other significant aspects of the Cassignol and Gillot technique are the omission of a ${}^{38}\text{Ar}$ spike, usually introduced to determine the absolute concentration of ${}^{40}\text{Ar}$, and adjustment of the air argon and sample argon volumes to a common argon pressure, or "working point" for the mass spectrometer.

Samples dated at the Centre des Faibles Radioactivités, France were measured using the procedures described in Cassignol, Gillot and Cornette [37,38]. Most of the same methods have been adopted for the argon measurements at Oregon State University. Specifically, we have regularly measured the isotopic composition of atmospheric argon with our AEI MS-10S mass spectrometer. Data from a period of 400 days (between successive filament changes) are shown in Fig. 6. The intervals during which experiments on the Icelandic samples were performed are also shown. Throughout this time R_a varied smoothly. Hence the uncertainty in the instrumental isotopic ratio for atmospheric argon at the time of measurement of the samples is of the order of 0.1% and samples with R_s exceeding the atmospheric composition by this small amount are considered to contain significant quantities of radiogenic ⁴⁰Ar. Repeated measurements of ⁴⁰Ar in aliquots of atmosphere and ³⁸Ar spike concentrations show that the instrument sensitivity is stable to 1%, so ⁴⁰Ar concentrations can be determined without spiking. This removes the correction for small amounts of ⁴⁰Ar and ³⁶Ar in the spike. We have measured R_a over an order of magnitude range in ⁴⁰Ar concentration, with no discernible variation. Hence we have not adjusted the size of the ⁴⁰Ar peak in air aliquots and samples to identical pressures.

Samples were provided from the paleomagnetic minicores from the outcrops described in the previous sections. These were free of surface weathering, compact (rather than vesicular), and contained rare, small olivine phenocrysts. Two to three cores from a single paleomagnetic site (50–75 g) were crushed to obtain the 0.5–1.0 mm size fraction. These chips were ultrasonically cleaned in distilled water and dried, then loaded for argon extraction. In one experiment (SK-1, second analysis) the sample was boiled in distilled water for 30 minutes, decanted and loaded immediately in an effort to reduce atmospheric contamination

TABLE 4

K-Ar ages of the excursion volcanic rocks in southwest Iceland, erupted during the Laschamp geomagnetic excursion

Sample No.	K	Radiogenic ⁴⁰ Ar	Radiogenic ⁴⁰ Ar	Age $\pm 1\sigma$
	(%)	$(\times 10^{-14} \text{ mole/g})$	(%)	(ka)
Oregon State University				
Siglubergshals (SK-1)	0.15	1.4668	1.24	56+21
	0.15	1.1384	0.61	44 + 23
Siglubergshals (SK-4)	0.15	1.3050	1.41	50 + 22
	0.15	1.3619	1.78	52 + 20
Siglubergshals (SK-13)	0.15	1.3479	0.65	52 ± 17
				$\mu = 51.4 \pm 9.0 \ (N = 5)$
Centre des Faibles Radioacti	vités			
Skalamaelifell	0.19	0.9136	0.35	26 ± 15
	0.19	1.3787	0.48	42 ± 18
	0.19	1.1794	0.50	36 ± 14
Bratthalskrokur	0.21	0.9801	0.132	27 ± 13
	0.21	1.2791	0.35	35 ± 15
	0.21	1.0133	0.30	28 ± 14
Siglubergshals	0.16	1.0299	0.19	37 ± 29
Hraunssels-Vatnsfell	0.19	1.4950	0.54	50 ± 14
(Vestara)	0.19	1.6611	0.58	52 ± 14
	0.19	1.9934	0.38	61 ± 25
Hraunssels-Vatnsfell	0.18	1.5615	0.52	50 ± 14
(Austara)	0.18	1.9269	0.58	63 ± 16
Höfdi	0.15	0.8140	0.19	31 ± 25
	0.15	1.3123	0.29	49 ± 25
				$\mu = 40.9 \pm 4.3 \ (N = 14)$
Weighted mean of all dated	Icelandic san	nples		$\mu = 42.9 \pm 3.9 \ (N = 19)$

Decay and abundance constants used in age calculations: $\lambda_{\epsilon} = 0.581 \times 10^{-10} \text{ yr}^{-1}$, $\lambda_{\beta} = 4.962 \times 10^{-10} \text{ yr}^{-1}$, ${}^{40}\text{K/K} = 1.167 \times 10^{-4}$. $\mu = \text{weighted mean of } N \text{ determinations. Uncertainties of } \mu \text{ are } 1\sigma \text{ of the mean.}$

from the sample surface. Samples were fused by RF induction heating and all active gases released were gettered on Ti-TiO₂ before the sample argon was admitted to the mass spectrometer. From 8 to 10 sets of ⁴⁰Ar and ³⁶Ar peaks were then collected by an automated peak-hopping computer algorithm. Fitted values from these data were used to compute the sample ratios R_s .

Age determinations from both laboratories are listed in Table 4. All samples have very low K-contents; from repeated measurements by atomic absorption the analytical uncertainty is less than 0.01% K. Samples from six of the transitional paleomagnetic field localities yielded ages in the range of 26–63 ka. Repeated measurements of the splits of the same rock are indicated. All 19 measurements are considered independent estimates for the purpose of calculating the mean age of the

geomagnetic excursion. Small but significant amounts of *⁴⁰Ar (0.2–1.8%) of total ⁴⁰Ar were detected, which led to large uncertainties in individual ages. However, the standard deviations for the mean of each laboratory's measurements are much smaller and the weighted means for the two sets are not significantly different. We are then justified in pooling all the measurements to obtain the weighted mean and its associated standard deviation, 42.9 ± 3.9 ka, as the age of the geomagnetic excursion recorded in these Icelandic rocks.

10. Discussion

The K-Ar ages of the excursion units at Skalamaelifell, Iceland, are very similar to the K-Ar determinations of the lavas that recorded the Laschamp excursion in the Massif Central in

TABLE 5

K-Ar and ⁴⁰Ar-³⁹Ar ages of volcanic rocks from the Massif Central, France, which recorded the Laschamp geomagnetic excursion

Sample No.		K (%)	Radiogenic 40 Ar (×10 ⁻¹³ mole/g)	Radiogenic ⁴⁰ Ar (%)	Age $\pm 1\sigma$ (ka)	
		()	(, , , , , , , , , , , , , , , , , , ,			
Oregon State University						
Laschamp		1.857	1.5770	2.76	49.0 ± 6.7	
Olby		1.733	1.1476	1.81	38.2 ± 6.3	
					$\mu = 43.3 \pm 4.6$ (N	= 2)
Hall and York (1978)						
Laschamp		weighted	mean of 2 K-Ar and 2 40	Ar- ³⁹ Ar ages	$\mu = 50.7 \pm 4.5$ (<i>N</i>	= 4)
Olby		weighted	mean of 8 K-Ar and 6 $^{\rm 40}$	Ar- ³⁹ Ar ages	$\mu = 47.2 \pm 1.6$ (<i>N</i>	=14)
Gillot et al. (1979)						
Laschamp		weighted	mean of 5 K-Ar ages		$\mu = 43.7 \pm 3.5$ (<i>N</i>	= 5)
Olby		weighted	mean of 5 K-Ar ages		$\mu = 49.9 \pm 4.0$ (<i>N</i>	= 5)
Weighted mean age of all Laschamp excursion loc	alities				$\mu = 46.6 \pm 1.2$ (N	= 30)

Decay and abundance constants used in age calculations: $\lambda_{\epsilon} = 0.581 \times 10^{-10} \text{ yr}^{-1}$, $\lambda_{\beta} = 4.962 \times 10^{-10} \text{ yr}^{-1}$, ${}^{40}\text{K/K} = 1.167 \times 10^{-4}$. Other definitions as in Table 4.

France. Hall and York [22] used a combination of K-Ar and ⁴⁰Ar-³⁹Ar analyses to date high-K lava flows from the Laschamp and Olby sites. Eliminating one extreme value, eighteen age determinations range from 20 to 62 ka with a weighted mean and standard deviation of $47.8 \pm$ 1.5 ka. Gillot et al. [23] reported ten K-Ar ages from the same flows, performed by methods similar to those described in Cassignol, Gillot and Cornette [37,38]. The weighted mean and standard deviation of the mean of these ages are 46.4 ± 2.6 ka. New K-Ar ages for these sites have also been determined at Oregon State University and are comparable with the two previous studies (Table 5). These sets of measurements performed at three laboratories on the same sample sites are indistinguishable and may be pooled to obtain a new estimate of the Laschamp excursion: 46.6 ka with a standard deviation of the mean of 1.2 ka for N = 30 determinations (Table 5). (Two standard deviations of the mean, $2\sigma = \pm 2.4$ ka, are nearly equivalent to 95% confidence limits, commonly used for comparisons in paleomagnetism and geochronology.)

The ages of the excursion units in southwestern Iceland and in the Massif Central, France, have overlapping one-standard-deviation confidence intervals on the means. Hence, there is probably no significant difference in the ages of the Skalamaelifell excursion units and the Laschamp and Olby flows, and based on their similar ages we suggest that they represent the same geomagnetic excursion. Levi and Karlin [25] have reported an excursion in rapidly deposited, late Quaternary sediments from DSDP Site 480 in the Gulf of California. Interpolation between age horizons assigned from δ^{18} O stratigraphy [39] suggests an age for the excursion of 49–51 ka, which they correlate with the Laschamp excursion.

During the Laschamp excursion the geomagnetic field was not a geocentric axial dipole, as indicated by the southeastern Pacific positions of the virtual geomagnetic poles (VGPs) of the excursion lavas from the Massif Central, France and southwestern Iceland (Fig. 7). The non-dipole character of the geomagnetic field during the Laschamp excursion is supported by the low paleointensity of the excursion units in southwestern Iceland ([33], this study) and France [24]. Furthermore, the 40° difference of the VGPs of the excursion units from southwest Iceland and France is consistent with significant non-dipole contributions to the excursion geomagnetic field. However, the difference in the VGPs might also



Fig. 7. VGPs associated with the excursion paleomagnetic directions in southwestern Iceland (circles) and Massif Central, France (squares). VGPs are indicated by closed symbols and site locations by open symbols.

Site	Site			PM direction		VGP		
	lat.	long.		D	Ι	lat.	long.	
Skalamaelifell	 + 63.9	-22.2	1. 15. 1	258	-15	-12.1	255.2	- 7
Laschamp [23]	+ 45.7	+2.9		237	- 66	- 51.9	247.7	

be due to (1) temporal evolution of the field and small age differences in the excursion units at the two locations, or (2) local/regional crustal magnetic anomalies, which might have contributed substantially to the paleomagnetic field during the low paleointensity excursion at either or both sampling areas.

The similar paleomagnetic directions of the Skalamaelifell excursion units, outcropping in an area approximately 10×10 km (Fig. 1), as well as their similar chemical and petrological affinities, suggest that the units were extruded during a short time interval. Because complete geomagnetic reversals require of the order of several thousand

years [4], we estimate that the sampled excursion units in southwest Iceland represent no more than several hundred years and possibly considerably less. This conclusion is further supported by the restricted range of elevation of the excursion units, which is of the order of 100 m, and relief buildup of this magnitude has been observed to occur in southwestern Iceland in less than 100 years. (Indeed, the time span is less than 10 years for the recent Surtsey and Heimaey eruptions, from late 1963 to 1967 and early to mid 1973, respectively.)

The Laschamp excursion was first reported 22 years ago [20], and the initial excitement of its discovery led to its inclusion as a subchron on

geomagnetic polarity time scales [40]. Failure to confirm the Laschamp elsewhere led to doubts about its global character and even its geomagnetic field origin was questioned. However, the Laschamp has regained excursion status through recent studies which demonstrated its geomagnetic field origin in considerable detail [24]. Its existence has now been observed in marine sediments in the Gulf of California [25], as well as in the Skalamaelifell lavas described here.

Recent advances in the K-Ar dating method make it possible to obtain ages for lavas as young as a few thousand years. Thus, combined paleomagnetic and geochronological studies of young lava sequences can be used, in principle, to characterize the global extent and behavior of late Pleistocene geomagnetic features such as, for example, the Mono Lake and Laschamp excursions.

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