J. Geomag. Geoelectr., **49**, 721–726, 1997

Intrinsic Bias in Averaging Paleomagnetic Data

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(Received February 20, 1996; Revised March 4, 1997; Accepted March 14, 1997)

The transformation of isotropically distributed geomagnetic poles to local site directions introduces slight apparent inclination shallowing if the directional average is compared to the geocentric axial dipole direction. This effect depends on site latitude and pole dispersions. For typical dispersions of poles, θ_{63} between 10° and 20°, the average inclination will appear too shallow by about 1° to 2° for site latitudes of 10° to 60° North or South. On the other hand, when some of the scatter might be due to isotropic scatter in directions, averaging all the data in polar space, will introduce a steepening effect. For most paleomagnetic studies this intrinsic inclination bias will be small in comparison with other uncertainties, typically of the order of 5° to 10°. However, for integrated high resolution studies of specific aspects of the geomagnetic field this methodological ambiguity might bias the results, and we caution against averaging paleomagnetic data only in directional space.

1. Introduction

The geomagnetic field resembles a geocentric axial dipole (GAD), when averaged over thousands of years (e.g., Opdyke and Henry, 1969). However, there is evidence for persistent non-dipole field components, including hemispheral asymmetries, often expressed as deviations of the inclinations from the GAD values. The magnitude of these persistent inclination anomalies is on the order of a few degrees and is latitude dependent (e.g., Wilson, 1970; Merrill and McElhinny, 1977; Schneider and Kent, 1990).

It is customary to use Fisher statistics (Fisher, 1953) to analyze paleomagnetic data. Such calculations of the average make the fundamental assumption that the data obey the isotropic Fisher distribution. The calculations can be done in both directional space and polar space, and there have been long term discussions in paleomagnetism whether the directions or the virtual geomagnetic poles (VGP) more suitably satisfy the isotropy condition of the Fisher statistics. Scatter arising from sampling, laboratory measurements, secondary overprinting, as well as local magnetic anomalies is commonly considered to be isotropic in directional space, while scatter caused by geomagnetic secular variation is assumed to be isotropic in VGP space (e.g., Irving, 1964). In reality, the scatter of paleomagnetic data is usually a combination of Fisher distributed poles and Fisher distributed directions.

The transformation of a dipole field to local directions is non-linear, and an isotropic distribution in one space is skewed in the other. It is well known that the dipole equation will transform an isotropic distribution of poles to an oval distribution of directions. The change in symmetry due to the dipole transformation from an isotropic distribution in one space to an oval distribution in the other space has received much attention in the literature and paleomagnetic textbooks (e.g., Cox, 1970; Tarling, 1971; McElhinny, 1973; McElhinny and Merrill, 1975; Merrill and McElhinny, 1983; Collinson, 1983; Tarling, 1983). However, there is no discussion in these references that the transformation might also introduce a systematic bias to the calculation of the average values.

In describing a different problem, Shuey *et al.* (1970) noted that there is an inclination error associated with the choice of data space, but they did not address the problem further. Creer (1983) showed

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that several types of simple synthetic field variations will result in an inclination error arising from the use of unit directional vectors. He does not present detailed evaluation of the magnitude of the apparent inclination shallowing due to this misuse of directions, but notes that this may play a significant role in estimating persistent non-dipole terms of the field.

To illustrate the problem of the biased averages, consider a site at magnetic latitude $\lambda_0 = 20°$. The corresponding dipole inclination I_0 can be calculated from the dipole equation, ($\tan I_0 = 2\tan\lambda_0$) which gives $I_0 = 36.1°$. Consider now isotropic scatter of poles. Two poles are 10° away from the rotation axis, along the site meridian, one towards our site and the other away from the site. This is equivalent to defining the two magnetic latitudes ($\lambda_0 + 10°$) and ($\lambda_0 - 10°$), which on average give λ_0. These two magnetic latitudes correspond to the inclinations ($I_0 + 13.1°$) and ($I_0 - 16.6°$) at our site. The average of these two inclinations is lower than I_0, and we have introduced 1.8° of inclination shallowing. Averaging of paleomagnetic directions, using Fisher statistics, when some of the scatter is of geomagnetic origin, will lead to anomalous shallow inclinations. Similarly, local noise averaged in polar space would introduce near-sided poles (inclinations that are too steep). Paleomagnetic data bases are possibly contaminated by such procedural biases, usually giving rise to shallow inclinations.

It is well known that the dipole equation is a non-linear transformation, and that somewhat different average values will be obtained depending on the sequence and space for calculating the averages. To the best of our knowledge, these differences have not been quantified, and paleomagnetists usually assume that they are sufficiently small to be ignored. Intuitively, one would expect that the magnitude of this biasing effect would vary with latitude and dispersion of the data, but we are unaware of a systematic, quantitative examination of this problem. In this study we construct synthetic distributions of geomagnetic poles to quantify the bias that can be introduced in paleomagnetic data when averaged only in directional space, a procedure that has been used by several workers.

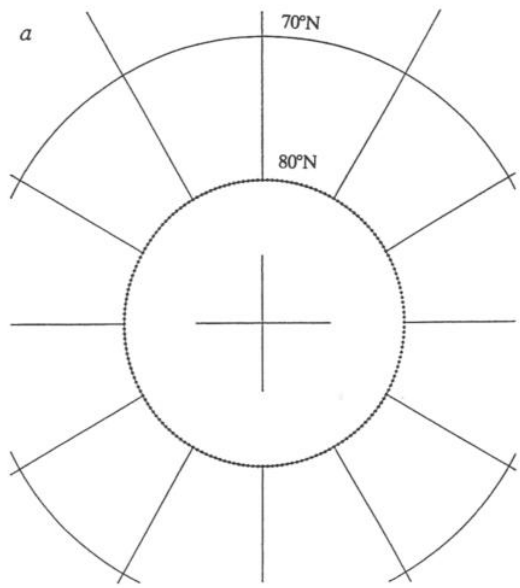
2. Simulations of this Study

To transform the Fisher distribution analytically from polar space, through the dipole equation and spherical trigonometry to a distribution of local directions is fairly complicated. For this simple study we chose to numerically transform a discrete isotropic distribution of poles to directions. The distribution of these directions turns out to be oval. Then we averaged the directions using Fisher statistics and compared the average to the expected GAD direction. These calculations were performed for various site latitudes and selected pole dispersions.

We generated two simple types of pole distributions: One consisted of a dipole precessing around the Earth's rotation axis, i.e. the poles were at a fixed latitude but various longitudes. The other consisted of a very large number of random-generated Fisher-distributed poles with known dispersion.

The dipole precession was generated by poles, at fixed latitudes of 70°, 75°, and 80°N. For each latitude we generated 180 poles in 2° longitude steps. The 180 pole positions were then transformed to directions at a site via the dipole equation and spherical trigonometry. The circle of poles around the rotation axis is transformed into an oval in directional space, and the center of the oval is noticeably removed from the GAD direction. An example of this effect is shown in Fig. 1. In Fig. 1(b) we show directions in a Hoffman projection, which he called (D', I') space (Hoffman, 1984). This is a stereographic projection of directions, where the directional sphere has been rotated such that the GAD direction is at the center. The Hoffman projection is ideal in comparing deviations from the GAD direction. It also reveals asymmetries (e.g. oval shape) about the GAD direction, since symmetric distributions about the GAD will be circular on the graph. The Fisher average of the skewed directions indicates that an apparent inclination shallowing was introduced, which depends on the site and pole latitudes. However, we note that the inclination shallowing is less than appears from the displacement of the oval, because of unequal data density on upper/lower part of the oval.

For a more realistic case, we generated Fisher-distributed poles with known dispersions. The main disadvantage of generating a large number of random Fisher-distributed poles is that a very large number



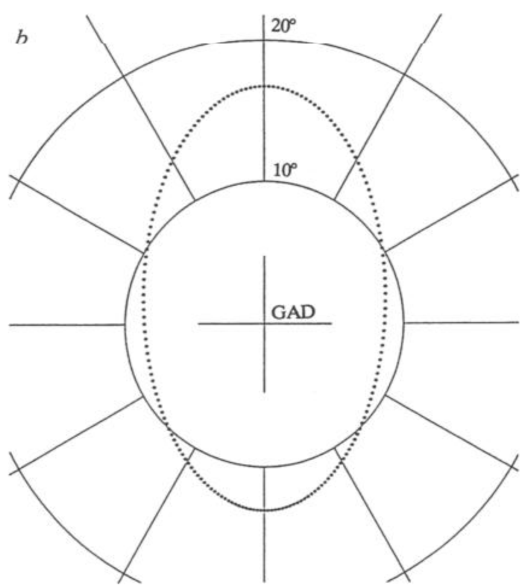


Fig. 1. Example of the data generated for the dipole precession. (a) Expanded view of the center of a stereographic polar projection. The centered cross represents the North Pole and the circles represent 70°, and 80°N latitude. A circle of 180 magnetic poles were generated at 80°N with 2° increments of longitude. (b) The corresponding 180 directions at site latitude of 20°N seen in a Hoffman projection (see text). The solid circles represent 10° and 20° deviation from the direction of the geocentric axial dipole (GAD), represented by the centered cross. The average of the 180 directions gives shallower inclination (higher on the graph) than the GAD value.

of poles is required to average out statistical fluctuations. Typically, on the order of thousands to millions of poles are needed. Calculations are needed at sufficient site latitudes for obtaining a continuous function of latitude, and for different values of pole dispersions. The calculations were performed for the angular standard deviations of the poles θ_{63} = 10°, 15°, and 20°. Thirty thousand Fisher-distributed poles were generated for every value of pole dispersion at each site latitude, which were varied from 0° to 90° in steps of 2°. The details of the random-pole generations are described in Arason (1991).

In Fig. 2 we show an example of Fisher-distributed poles that are skewed when transformed into directional space. Not only are the circles in polar space (Fig. 2(a)) transformed to ovals in directional space (Fig. 2(b)), but these ovals are visibly displaced upwards from the GAD inclination, indicating on average an inclination shallowing effect. The circles in Fig. 2(a) encircle 10%, 20%, 30%, ..., 80%, and 90% of a Fisher distribution with dispersion, θ_{63} of 20°. These nine circles were transformed to directions in Fig. 2(b) in a Hoffman projection. Note that both the ovality and the center displacement increase for the larger circles.

The results of this study are summarized in Fig. 3, where we show the apparent inclination shallowing for circularly distributed poles, when Fisher-averaged as directions. The inclination shallowing is shown as a function of site latitude for the two simple types of pole distributions at various pole dispersions. These

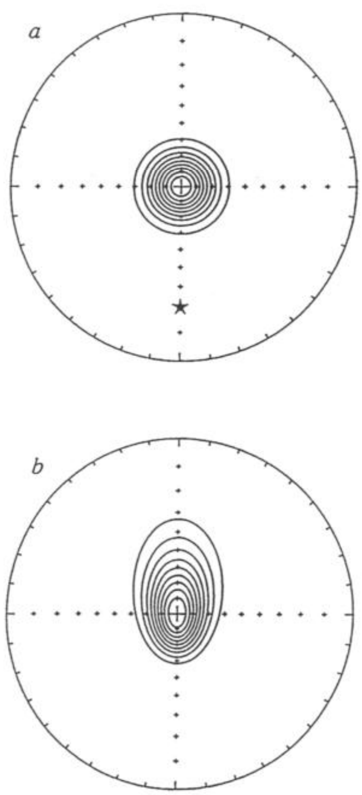


Fig. 2. Stereographic projections of Fisher distributed polar data. (a) The nine circles represent θ_{10}, θ_{20}, ..., θ_{90}, (θ_{90} encircles 90% of the distribution) of a Fisher distribution with θ_{63} = 20° on a pole centered stereographic projection. The star represents a site at 20°N. (b) the nine circles have been transformed to the directional space seen at 20°N in a Hoffman projection. The interval between crosses is 10° of latitude in (a) and 10° of direction away from GAD in (b). The average direction is biased towards a shallower inclination (higher on the graph) than the GAD inclination.

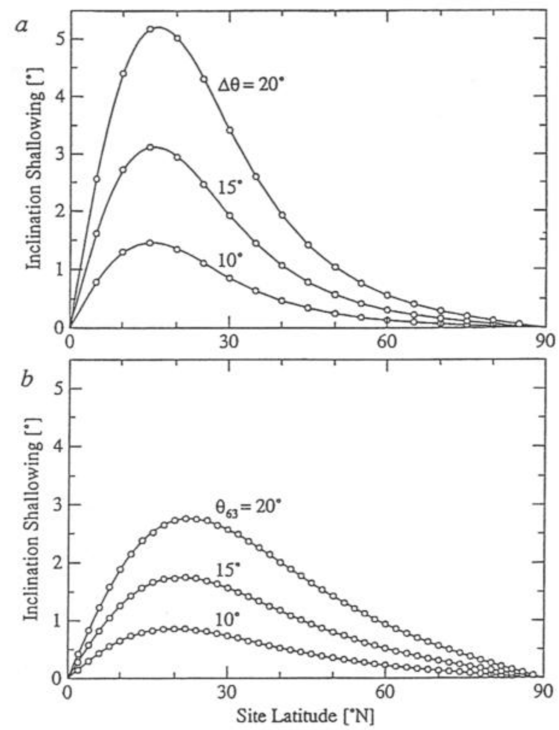


Fig. 3. The inclination shallowing obtained by averaging isotropic polar data in directional space as a function of site latitude. (a) Results of the dipole precession about the rotation axis, with pole-latitude deviations of $\Delta\theta = 10°$, 15°, and 20°. (b) Results of random-generated Fisher-distributed poles for pole dispersions of $\theta_{63} = 10°$, 15°, and 20°. This simple study shows that Fisher-averages of directions where the geomagnetic poles are isotropically distributed, will introduce an inclination shallowing effect of a few degrees.

graphs show that the proposed biasing effect increases with pole dispersion, and can account for about 1° to 2° of inclination shallowing for site latitudes 10° to 60° North or South.

3. Discussion

The angular dispersion of virtual geomagnetic poles due to secular variation is latitude dependent with angular standard deviation, θ_{63}, in the range of 10° to 20° (Merrill and McElhinny, 1983). For such dispersions an apparent inclination shallowing or far-sided poles can be introduced by doing all the Fisher-averages in directional space. The apparent inclination anomaly is on the order of 1° to 2° for site latitudes of 10° to 60°, see Fig. 3(b). Some paleomagnetic studies averaged the data in directional space only, even for rock units from several sites (see for example a review by Harrison and Lindh (1982)).

Harrison (1980) showed that virtual geomagnetic poles do not obey the Fisher distribution exactly. The true distribution of the virtual geomagnetic poles derived from some Icelandic lavas is better described by a mixture of 10% random distribution and 90% Fisher distribution. Such a deviation from the Fisher distribution will increase the inclination shallowing effect described in this study.

Although, the inclination shallowing effect proposed by this study is rather small, there are other procedures and processes that may also lead to an accumulation of inclination shallowing biases. For example, sample shape can enhance apparent inclination shallowing (Steele, 1989). Furthermore, random-undetected block rotations will favour shallow inclinations (e.g., Arason and Levi, 1990, model 3b; Calderone and Butler, 1991).

Therefore, some inclination shallowing bias probably exists in large data sets that are used to estimate apparent polar wander paths and persistent non-dipole components of the geomagnetic field, including hemispheral asymmetries. On the other hand, averaging local scatter in polar space might introduce false inclination steepening in the data. Ultimately, it is difficult and somewhat ambiguous to decide which space is geophysically appropriate, because usually the data scatter is a mixture of noise from both polar and directional spaces in unknown proportions.

The authors are grateful for critical comments by Gary Egbert.

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