IUGG 2007 – ASI009-3
International Union of Geodesy and Geophysics, XXIV General Assembly, 2-13 July 2007, Perugia, Italy

The Maximum Likelihood Solution for Inclination-only Data

Arason, P
arason@vedur.is
The Icelandic Meteorological Office, Bustadavegi 9, IS-150 Reykjavik, Iceland

Levi, S
Shaul_Levi@msn.com
College of Oceanic and Atmospheric Sciences, Oregon State University, Corvallis, OR 97331 United States

The arithmetic means of inclination-only data are known to introduce a shallowing bias. Several methods have been proposed to estimate unbiased means of the inclination along with measures of the precision. Most of the inclination-only methods were designed to maximize the likelihood function of the marginal Fisher distribution. However, the exact analytical form of the maximum likelihood function is fairly complicated, and all these methods require various assumptions and approximations that are inappropriate for many data sets. For some steep and dispersed data sets, the estimates provided by these methods are significantly displaced from the peak of the likelihood function to systematically shallower inclinations. The problem in locating the maximum of the likelihood function is partly due to difficulties in accurately evaluating the function for all values of interest. This is because some elements of the log-likelihood function increase exponentially as precision parameters increase, leading to numerical instabilities. In this study we succeeded in analytically cancelling exponential elements from the likelihood function, and we are now able to calculate its value for any location in the parameter space and for any inclination-only data set, with full accuracy. Furtermore, we can now calculate the partial derivatives of the likelihood function with desired accuracy. Locating the maximum likelihood without the assumptions required by previous methods is now straight forward. The information to separate the mean inclination from the precision parameter will be lost for very steep and dispersed data sets. It is worth noting that the likelihood function always has a maximum value. However, for some dispersed and steep data sets with few samples, the likelihood function takes its highest value on the boundary of the parameter space, i.e. at inclinations of ±90°, but with relatively well defined dispersion. Our simulations indicate that this occurs quite frequently for certain data sets, and relatively small perturbations in the data will drive the maxima to the boundary. We interpret this to indicate that, for such data sets, the information needed to separate the mean inclination and the precision parameter is permanently lost. To assess the reliability and accuracy of our method we generated large number of random Fisher-distributed data sets and used seven methods to estimate the mean inclination and precision paramenter. These comparisons are described by Levi and Arason at the 2007 IUGG meeting. The results of the various methods is very favourable to our new robust maximum likelihood method, which, on average, is the most reliable, and the mean inclination estimates are the least biased toward shallow values. Further information on our inclination-only analysis can be obtained from:
http://www.vedur.is/~arason/paleomag