

# Comparisons of Inclination-only Statistical Methods

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## 1 Abstract

Paleomagnetic data from borecores often lack declinations, and the arithmetic means of inclination-only data are known to introduce a shallowing bias. Several methods have been proposed to estimate unbiased means of the inclination along with measures of the precision.

Using maximum likelihood estimates, we were able to derive a robust technique of inclination-only statistics for the mean inclination and precision parameter, without making the assumptions and approximations of previous methods. Our method was described by **Arason and Levi** (2006).

To assess the reliability and accuracy of our method, we generated random Fisher-distributed data sets and used seven methods to estimate mean inclinations and precision parameters. We used true inclination values of 0, 10, ..., 80, and 90 degrees; true precision parameters of 10, 20, 40, and 100; the sample number in each data set was 5, 10, 20, and 100.

For each combination, we generated one thousand random Fisher-distributed data sets, and for these 160 000 data sets we calculated the true Fisher mean, also using declinations. For inclination-only data the mean was calculated using the following methods: Arithmetic mean; **Kono** (1980); **McFadden-Reid** (1982), using both their original and modified methods; **Enkin-Watson** (1996) gaussian-estimates; finally, we obtained maximum likelihood estimates by our new robust technique.

In many cases the estimates provided by the previous methods are significantly displaced from the true peak of the likelihood function to systematically shallower inclinations, especially for steep and dispersed data. It appears that the mean inclination estimates of the original McFadden-Reid statistics, still used by some paleomagnetists, is nearly identical to the arithmetic mean, and, in our opinion this method should be abandoned.

Comparisons of the results of the various methods is very favourable to our new maximum likelihood method. On average, it gives the most reliable estimates and the mean inclination estimates are the least biased toward shallow values. Further information on our inclination-only analysis, method, and program codes can be obtained from:

<http://www.vedur.is/~arason/paleomag>

## 3 The Importance of Unbiased Mean Inclinations

Over forty years ago **Briden and Ward** (Pure Appl. Geophys., 63, 133-152, 1966) showed that for Fisher-distributed inclination-only data, the arithmetic mean is biased toward shallow inclinations.

In paleomagnetic applications this inclination bias is usually less than a few degrees. For individual studies such a discrepancy is of a minor importance and usually well within the confidence limits of the study. However, since this is a one sided bias, any attempt to combine results of many studies, in order to increase resolution, may lead to errors. For example, studies of long term non-dipole terms in the geomagnetic field require averaging the mean inclinations of many studies, indicating a long term effect of 1-2°. Improper procedures for estimating mean inclinations in individual studies can seriously affect such estimates.

For the 160 000 random data sets, we compared average inclination shallowing bias versus true inclination for various values of ( $\kappa$ ). The Arithmetic mean and the McFadden-Reid methods give the worst biases. The Arason-Levi method results in the least biased estimates, and for  $\kappa > 40$  it results in insignificant mean bias for inclinations up to 80°.

We have compared the results of various methods for the original numerical example presented by **Fisher** (Proc. R. Soc. London, Ser. A, 217, 295-305, 1953). The Briden-Ward, Kono and McFadden-Reid methods all attempt to evaluate the maximum likelihood estimates. For this data set, only the Arason-Levi maximum likelihood method accurately calculates the location of the maximum value of the likelihood function.

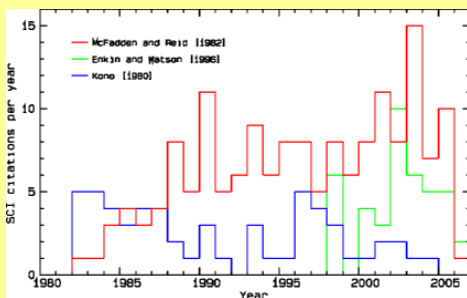
The information to separate the mean inclination from the precision parameter will be lost for very steep and dispersed data sets. For such data sets the likelihood function has a tendency to take its maximum value on the edge, i.e. at  $I = \pm 90^\circ$ . However, the Enkin-Watson method will tend to give an artificial solution to such data sets with lower inclinations and very low precision. Such solutions depend critically on the assumptions of the calculation method, and we feel that in such cases it may be better for scientists to have a clear indication that the information to separate ( $I, \kappa$ ) is permanently lost.

## 2 The Studied Methods

**Kono** (J. Geophys. Res., 85, 3878-3882, 1980) presented a method based on equating the expectation values of  $\cos \theta$  and  $\cos^2 \theta$  of the distribution to the data. In principle this is a correct method of moments estimation, and asymptotically unbiased as  $N \rightarrow \infty$ . For some examples of steep and dispersed inclinations the method breaks down.

**McFadden and Reid** (Geophys. J. R. Astron. Soc., 69, 307-319, 1982) criticized the assumptions of Kono and suggested a different solution to the maximum likelihood problem. Unfortunately there was an error in one of their equations. Although this error was early on modified by several workers, some paleomagnetists are still using the erroneous original method. Even with the correction the McFadden-Reid method is based on approximations that turn out to be inappropriate for dispersed steep inclinations and lead to inclination biases.

**Enkin and Watson** (Geophys. J. Int., 126, 495-504, 1996) presented a new approach. They weighted the likelihood function with a Bayesian factor, so their method is not solution to the maximum likelihood problem. This weighing gives better constraints of the solution for very dispersed and steep inclinations. They presented three totally different methods depending on the dispersion and steepness of the data. These are 1) arithmetic mean, 2) gaussian estimate, 3) marginal likelihood. In this comparison we have focused on their gaussian estimation, but for the steepest data one should use the marginal likelihood. However, the marginal likelihood leads to increased inclination bias as they change to asymmetric confidence intervals.



The number of Science Citation Index (SCI) citations indicates that the most favourite inclination-only method among scientists may be the McFadden-Reid method.

**Arason and Levi** (Eos Trans. AGU, 87, GP21B-1312, 2006) have outlined a new method that directly evaluates the maximum likelihood estimates of mean inclination and precision parameter without previous assumptions. Evaluation of the likelihood function and its derivatives is very problematic. For ordinary paleomagnetic data the direct evaluation of these functions includes exponential elements that lead to an overflow in any ordinary programming language. We were successful in analytically cancelling these exponential elements from the functions, and by accurate evaluation of Bessel and other functions we are able to accurately calculate the location of the maximum of the likelihood function.

## 4 Conclusions

In this study we compared methods to estimate mean inclinations from inclination-only data.

Independent of methods, the paleomagnetic inclination shallowing bias of individual studies is usually well within the confidence interval of the mean. However, it is essential to estimate unbiased inclination means from inclination-only data, because the results are sometimes combined from several individual studies to increase resolution.

Unbiased estimates of other statistical parameters are usually not as important as the mean inclinations.

The comparisons of this study shows that the McFadden-Reid method, which appears to be widely used, often leads to significant inclination biases. The original method includes an error and appears to give mean inclinations almost identical to the arithmetic mean. A modification to the method is not much better. In our opinion the McFadden-Reid method should be abandoned.

**Arason and Levi** (2006) presented a new method, which accurately calculates the maximum likelihood estimates of mean inclinations from inclination-only data. This study indicates that this method results in the least inclination biases.

Detailed information on our inclination-only comparisons, analysis, method, and program codes can be obtained from:

<http://www.vedur.is/~arason/paleomag>